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A Study on the Feasibility of Measuring the Emissivity with the Laser-Flash Method

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Abstract The laser-flash method is a fast, widely used and well established technique to measure the thermal diffusivity. Since its introduction in the 1960s, it was proposed to expand this technique to the measurement of heat capacity and emissivity. Currently, the measurement of spectral emissivity at high temperatures is connected with relatively large uncertainties, although the spectral emissivity is an essential parameter for applications, e.g., in the lamp industry and fusion research. In this work, a theoretical study is presented on the possibility of emissivity measurements using the laser-flash method. Two mathematical approaches are discussed which solve the problem, that a measured temperature rise—necessary to calculate the emissivity—itself depends on the emissivity. It is shown that both methods have a negligible arithmetic error, making them applicable to be used in future work.

Keywords Emissivity · High temperature · Laser-flash method · Pulse heating · Radiation thermometry

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1 Introduction

The spectral emissivity is an important thermophysical material parameter for both radiation thermometry and thermal simulations, as non-contact temperature measurements are based on the calculable blackbody radiation according to Planck's law of thermal radiation. For real bodies and surfaces, the spectral emissivity has to be taken into account. Therefore, this material parameter is as important as the exact calibration of the radiation thermometer.

For instance, the lighting industry requires very accurate information on the temperature of the heated tungsten filaments and the electrodes of incandescent and discharge lamps, respectively, to optimize their efficiency and lifetime. The working temperature in these devices typically exceeds 2800 K. Especially for incandescent lamps, the lifetime crucially depends on the filament temperature, as a 1 % higher temperature causes a decrease in lifetime of 25 % [1].

The spectral emissivity is also an important parameter for numerical simulations and the design of high-temperature applications, such as power plants, turbines, and aerospace systems. Also, research for novel fusion reactors requires well-known material parameters at high temperatures in order to simulate and design the combustion chamber. In addition to the emissivity, other material parameters such as specific heat capacity and thermal conductivity are also required.

In spite of its relevance in the high-temperature range above 1800 K, there are no reliable standard measurement techniques for emissivity and, therefore, only a few data is available. One of the most frequently investigated high-temperature materials is tungsten, because of its importance for the lighting industry and nuclear power plants. The most often cited reference on the emissivity of tungsten from de Vos et al. [2] dates from the early 1950s, and comparison to other literature data reveals a variation of the values from various references typically in the 10 % range [3]. These variations cannot only be attributed to measurement uncertainties, but also—and more importantly—to sample variation. In the case of metals such as tungsten, the manufacturing process, for instance, has a significant influence on the spectral emissivity [4].

The above-mentioned problems underpin the necessity for a practical method, which allows a frequent measurement of the emissivity at high temperatures on the respective sample used.

Several methods have been proposed and applied to measure the emissivity, mostly based on a static measurement principle [5–11]. These techniques comprise a reference radiation source with a known emissivity and known temperature, which is usually a blackbody with an emissivity close to unity. The spectral radiance of this blackbody with a known temperature is then compared with the radiance emitted by the sample under investigation, for which the surface temperature also has to be known. Details and limitations of this method are described in Sect. 2. Due to the described limitations, this method is limited to temperatures below approximately 1800 K. To measure emissivities at higher temperatures up to above 3000 K, a novel, dynamical method is proposed, which is described in detail in Sect. 3. The theoretical background is derived in Sect. 4, while Sect. 5 gives a discussion and an outlook for implementing this novel method.

2 Conventional Method to Measure the Spectral Emissivity

The directional spectral emissivity $\varepsilon(\lambda)$ is a dimensionless quantity and is, for a radiating sample, defined as the ratio of the spectral radiance $L_{\lambda,\text{Sample}}$ of the sample to the spectral radiance of an ideal blackbody $L_{\lambda,\text{BB}}$ at the same temperature

$$\varepsilon(\lambda) = \frac{L_{\lambda,\text{Sample}}\,(T,\lambda)}{L_{\lambda,\text{BB}}\,(T,\lambda)}. \tag{1}$$

This principle is used for the static emissivity measurement, a technique, which is shortly described in the following, referring to the setup used at the PTB [5,6]. A schematic of the experimental setup is shown in Fig. 1. The emissivity measurements are performed as a spectrally resolved comparison of the sample under test and a reference blackbody by using a Fourier-transform spectrometer. The measurement principle follows the definition of the emissivity in Eq. 1. The emissivity of a sample is determined by measuring the spectral radiant power emitted by the sample with respect to the spectral radiant power emitted by a reference blackbody. Due to the constant geometry of detection, this is equivalent to the determination of the ratio of the spectral radiances.

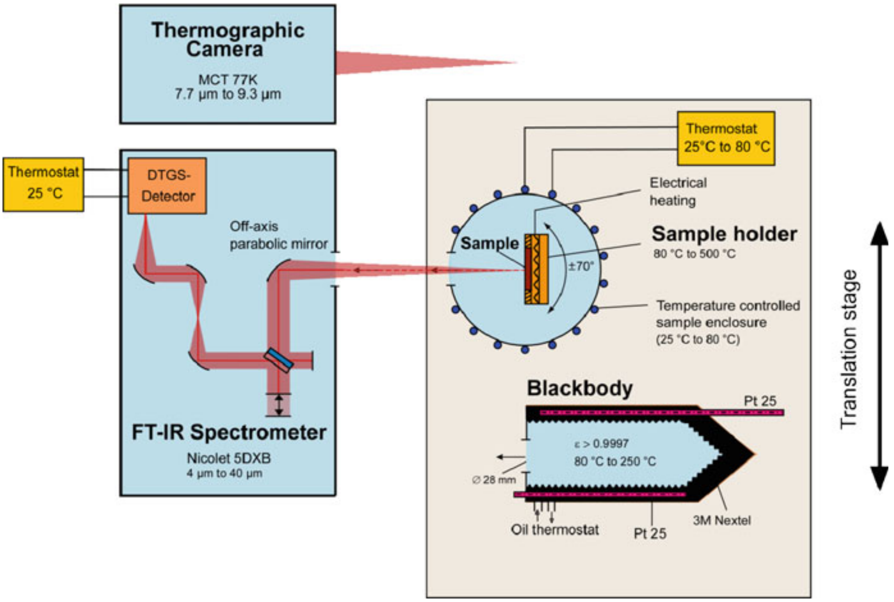

Fig. 1 Experimental setup for directional spectral emissivity measurements under air at PTB. It consists of a Fourier-transform spectrometer, a sample holder, and a reference blackbody radiator, which are all temperature stabilized. The sample holder with the sample and the reference blackbody can be positioned in the measurement area of the spectrometer and in front of a thermographic camera

Both measurements have to be done sequentially. For emissivity measurements under air, the sample and the blackbody are alternately positioned in the detection area of the Fourier-transform spectrometer with the help of a translation stage.

The emissivity of a sample is smaller than unity: therefore, the sample also reflects part of the radiation emitted by the surroundings toward the spectrometer. Using a hemispherical enclosure at fixed temperature as an isothermal surrounding and with constant spectral emissivity (compare Fig. 1), the ambient radiation incident on the sample can be calculated and consequently the emissivity of the sample can be determined.

A precise knowledge of the temperature of the emitting surface and of the blackbody is mandatory for calculation of the directional spectral emissivity. The procedure for its calculation is described in detail in [5]. Briefly, the thermal conductivity of the sample and of a possible sample coating as well as their thicknesses have to be known. The overall heat loss from the sample surface by radiation and convection has to be considered. Both can be calculated for an estimated hemispherical emissivity and an estimated surface temperature of the sample. Balancing the overall heat loss with the heat flux through the sample allows the iterative determination of the surface temperature. In a second "outer" iteration, the estimated hemispherical emissivity is refined by integrating the emissivities at the obtained surface temperature (the result of the first iteration), an improved value of the emissivity is found which serves as a new starting value for the next iteration. After a few steps, both iterations converge. This static method is realized in different setups, as reported in the literature [7–10].

The surface-temperature measurement limits the application of the above-mentioned method, as it is critically influenced by convective heat losses. Performing this method under vacuum prevents the convective heat losses and allows considerably higher temperatures [11]. However, the contact thermometers used for the surface-temperature measurements, usually platinum resistance thermometers or thermocouples, are prone to significant drifts at temperatures above 1300 K. As the surface temperature of the sample has to be known, these convection and temperature sensor instabilities set an upper limit to the static emissivity measurements to about 1800 K.

Some modifications of the static approach have been described in the literature, which try to overcome these limitations. An interesting approach is described in [12]. Here, the sample forms the bottom of the blackbody used as the reference source. After measuring the blackbody radiance, the sample is pushed forward immediately to the entrance of the blackbody, where the sample radiance is measured. Due to the assumed adiabatic movement, it is assumed that the surface temperature of the sample does not change during the fast movement. However, this assumption and the field of view problem of the radiance measuring device are the main uncertainty contributions to this innovative method.

In other approaches, the surface temperature of the sample is measured using small drill holes with increased emissivity. However, due to the depth of the drill holes, only an integrated temperature over the depth of the hole is measured, significantly limiting the uncertainty of the obtained surface temperature in particular at higher temperatures [13–15].

3 Dynamic Emissivity Measurement Using the Laser-Flash Technique

Already in the first description of the flash method [16], the possibility of measuring several thermophysical properties with one experiment has been stressed. Heating of a sample by an optical pulse results in an increase of the sample temperature, which is recorded at the opposite site. Using simply the relative time evolution of the temperature rise without its absolute value, as is done by the usual laser-flash setups, enables the determination of the thermal diffusivity. However, using the time evolution and the absolute height of the temperature step, together with a knowledge of the absorbed optical power yields the possibility to measure the thermal diffusivity, heat capacity, and with it, the thermal conductivity. Measuring the heat capacity requires a knowledge of the absorbed power and therefore the energy of the optical pulse and the sample emissivity. Vice versa, with a known heat capacity, the emissivity can, in principle, be determined. Despite the proposed flexibility, the flash method is usually restricted to the measurement of the thermal diffusivity. The fundamental reason for this limitation is the problem of an absolute measurement of the temperature rise and the power of the incoming optical radiation with the required low uncertainty.

For measuring the heat capacity or the emissivity, the following straightforward equation can be used

$$\varepsilon(\lambda, T)E_\lambda = mc_p(T)\Delta T. \tag{2}$$

The left side of Eq. 2 represents the absorbed optical power (keep in mind that the absorptivity is equal to the emissivity $\varepsilon(\lambda, T)$), while the right side is the increase in internal energy of a sample with mass m and specific heat $c_p(T)$ after a step in temperature of ΔT. It is evident from Eq. 2 that a knowledge of the optical flash energy E_λ and the temperature step ΔT allows a measurement of $\varepsilon(\lambda, T)$ if $c_p(T)$ is known, or a measurement of $c_p(T)$ if $\varepsilon(\lambda, T)$ is known. Of course, either the emissivity or the specific heat measurement requires a knowledge of the mass m of the sample, but this is a straightforward measurement. Both the measurement of the incoming laser energy and the absolute measurement of the temperature rise are demanding tasks and up to now prevent the application of the flash method to absolute measurements of specific heat or emissivity. An alternative approach using the flash method has been used for the measurement of the specific heat, which relies on relative measurements using a sample with a known specific heat and emissivity as a reference and is, therefore, not an absolute method [17]. At the Physikalisch-Technische Bundesanstalt (PTB), the national metrology institute of Germany, the measurement of laser energy and absolute temperature can be performed with state-of-the-art uncertainties [18–20]. This expertise takes advantage of a commercial laser-flash apparatus for in situ measurements of the laser energy incident to the sample and the resulting temperature rise.

Figure 2 shows the principle design of the laser-flash apparatus for dynamic emissivity measurements. It basically consists of a high energy pulsed laser, a high-temperature furnace which contains the sample, and a radiation thermometer to measure the temperature rise of the sample. In contrast to a conventional laser-flash apparatus used for thermal-diffusivity measurements, the absolute laser energy is

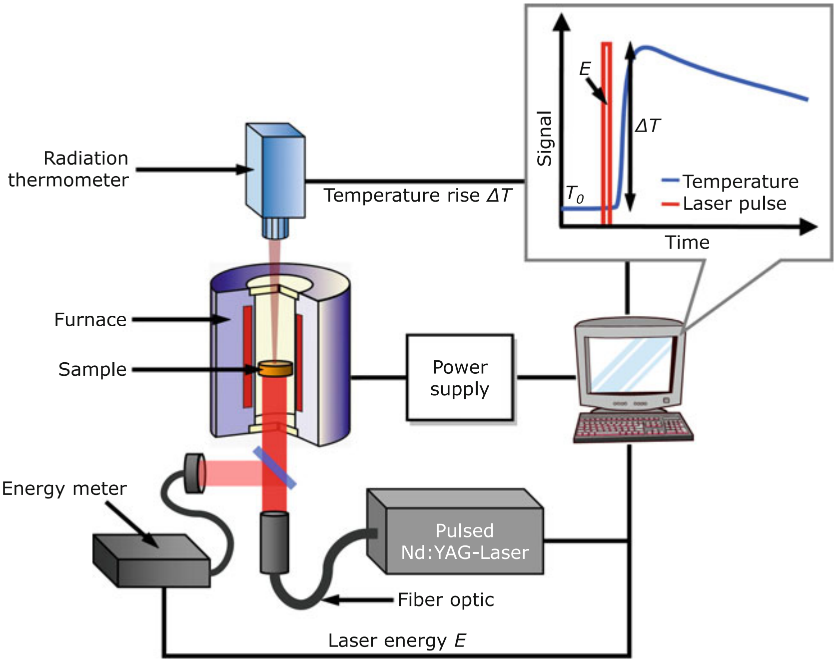

Fig. 2 Schematic of the dynamic emissivity measurement with a laser-flash apparatus. Inset shows the principle measurement data to calculate the spectral emissivity: the energy of the laser pulse E and the temperature rise, with the starting temperature T_0 and the absolute value of the temperature rise ΔT. Note that for a standard laser-flash measurement only the time evolution of the temperature is necessary to calculate the thermal diffusivity

measured via a beam splitter and a laser energy meter. Also, the relative measuring infrared detector is replaced by a calibrated radiation thermometer for measuring the absolute value of the temperature rise.

A detailed experimental description of the apparatus will be the topic of a future publication. Here, an in-depth analysis of the principal feasibility of such an approach for the measurement of spectral emissivity at higher temperatures will be given.

The spectral distribution of the spectral emissivity can be obtained either using an absolute measuring spectrometer or with a combination of a radiation thermometer with narrow bandwidth filters around the laser wavelength λ_0 and a relative measuring spectrometer. With both configurations, the emissivity at the laser wavelength $\varepsilon(\lambda_0, T_0)$ can be calculated as described in Sect. 4, and with it, the true surface-temperature T_0. The spectral distribution of the emissivity can be calculated with Planck's law of thermal radiation at T_0 and the normalized spectrum obtained with the spectrometer as shown in

$$\varepsilon(\lambda, T_0) = \underbrace{\left[\frac{L_\lambda(\lambda, T_0)}{L_\lambda(\lambda_0, T_0)}\right]}_{\text{normalized spectrum}} \frac{L_{\lambda,\mathrm{BB}}\,(\lambda_0, T_0)}{L_{\lambda,\mathrm{BB}}\,(\lambda, T_0)}\varepsilon(\lambda_0, T_0). \tag{3}$$

4 Theoretical Discussion of Measurement Principle

As mentioned in Sect. 3, a simple equation can be derived for the spectral emissivity $\varepsilon(\lambda)$ under the conditions of a laser-flash experiment. Starting from the definition of the heat capacity, with the laser energy at a certain wavelength E_λ, the mass m and the specific heat capacity c_p of the sample together with the resulting temperature rise ΔT

$$\varepsilon(\lambda, T) = \frac{mc_p(T)\Delta T(T)}{E_\lambda}. \tag{4}$$

However, the temperature rise ΔT_M measured with the radiation thermometer itself depends on the emissivity and the spectral sensitivity of the radiation thermometer s_λ

$$\Delta T_\mathrm{M} = \Delta T_\mathrm{M}(\varepsilon, s_\lambda, T). \tag{5}$$

Consequently, the calculation of the emissivity via ΔT_M seems to be a circular argument. This can be overcome either by iteration of ΔT_M (see Sect. 4.1) or by deriving an implicit equation from Planck's law of thermal radiation (see Sect. 4.2).

In both cases, it is essential that the sample top and bottom surfaces show quasi-identical absorbance and, therefore, emissivity. First of all, the texture of both surfaces has to be quasi-identical and second, due to the spectral variation of the emissivity, the spectral sensitivity of the radiation thermometer s_λ has to be in a narrow bandwidth region around the laser wavelength λ_0. This can be mathematically expressed with the Dirac delta function $\delta(\lambda - \lambda_0)$ as

$$\begin{aligned} E_\lambda &\approx E_0\delta(\lambda - \lambda_0); \quad E_0 = \text{const.} \\ \Rightarrow s_\lambda &\approx s_0\delta(\lambda - \lambda_0); \quad s_0 = \text{const.} \end{aligned} \tag{6}$$

Equation 6 defines the laser energy E_0 and the spectral sensitivity of the radiation thermometer s_0 at the laser wavelength λ_0. Both can be considered constant for the following considerations.

For the sake of completeness, it should be noted that first the heat transport has to be one-dimensional and quasi-adiabatic, ensuring that all of the absorbed laser energy contributes to the temperature rise on the backside of the sample. Due to the fast process and the sample geometry (thin disk), this requirement is satisfied. Second, the temperature rise ΔT has to be small compared to the sample temperature T_0, assuring that the temperature dependence of the emissivity and the specific heat capacity can be neglected and both can be considered as constant

$$\begin{aligned} \varepsilon(\lambda, T) &= \varepsilon(\lambda_0, T_0) \equiv \varepsilon_0 = \text{const.} \\ c_p(T) &= c_p(T_0) \equiv c_{p,0} = \text{const.} \end{aligned} \tag{7}$$

4.1 Iteration

The basic concept, of an iterative solution of the implicit problem in Eqs. 4 and 5 to calculate the emissivity, is the recalculation of the real temperature rise ΔT_{i+1} with the previous calculated emissivity ε_i, the furnace temperature T_0, and the measured temperature rise ΔT_{M}

$$\begin{aligned} \varepsilon_i(\lambda_0, T_0) &= \frac{mc_{p,0}\Delta T_i}{E_{\lambda_0}}, \\ \Delta T_{i+1} &= \Delta T_{i+1}\left(T_0, \Delta T_{\mathrm{M}}, \varepsilon_i\right). \end{aligned} \tag{8}$$

ΔT_{M} already gives a rough estimate for the real temperature rise ΔT and is the starting point for the iteration

$$\Delta T_{i=0} = \Delta T_{\mathrm{M}}. \tag{9}$$

The recalculation is based on the fundamental relation between the measured temperature T_{M}, which is associated with the spectral radiance L_λ of the sample, and the blackbody temperature, which gives the correct value for ΔT

$$L_\lambda\left(\lambda_0, T_{\mathrm{M}}\right) = \varepsilon_i L_{\lambda,\mathrm{BB}}\left(\lambda_0, T_{i+1}\right). \tag{10}$$

The spectral radiance of a blackbody $L_{\lambda,\mathrm{BB}}$ can be calculated using Planck's law of thermal radiation (Eq. 11), or using Wien's approximation to Planck's law (Eq. 12), where c_1 and c_2, respectively, are the first and second radiation constant in Planck's law

$$L_{\lambda,\mathrm{BB}}^{\mathrm{Planck}}\left(\lambda_0, T\right) = \frac{c_1}{\lambda_0^5}\left[\exp\left(\frac{c_2}{\lambda_0 T}\right) - 1\right]^{-1}, \tag{11}$$

$$L_{\lambda,\mathrm{BB}}^{\mathrm{Wien}}\left(\lambda_0, T\right) = \frac{c_1}{\lambda_0^5}\exp\left(-\frac{c_2}{\lambda_0 T}\right). \tag{12}$$

Using Planck's law results in an implicit equation for the temperature rise, which has to be solved numerically in each iteration step according to

$$\begin{aligned} \Delta T_{i+1}^{\mathrm{Planck}} &= (T + \Delta T)_{i+1} - T_{i+1}, \text{ with} \\ \varepsilon_i &= \frac{\exp\left(\frac{c_2}{\lambda_0 T_{\mathrm{M}}}\right) - 1}{\exp\left(\frac{c_2}{\lambda_0 T_{i+1}}\right) - 1} \quad \text{and} \quad \varepsilon_i = \frac{\exp\left(\frac{c_2}{\lambda_0 (T_{\mathrm{M}} + \Delta T_{\mathrm{M}})}\right) - 1}{\exp\left(\frac{c_2}{\lambda_0 (T + \Delta T)_{i+1}}\right) - 1}. \end{aligned} \tag{13}$$

Using Wien's approximation (Eq. 12) yields an explicit equation for the temperature rise for each iteration step according to

$$\Delta T_{i+1}^{\mathrm{Wien}} = \left[\frac{1}{T_{\mathrm{M}} + \Delta T_{\mathrm{M}}} + \frac{\lambda}{c_2}\ln\varepsilon_i\right]^{-1} - \left[\frac{1}{T_{\mathrm{M}}} + \frac{\lambda}{c_2}\ln\varepsilon_i\right]^{-1}. \tag{14}$$

Therefore, one can choose between an explicit and implicit approach to calculate the temperature rise. The iteration with Eq. 14 is prone to an error inherent to Wien's approximation of Planck's law, while the more complex iteration with the implicit Eq. 13 has no analytical error in the single iteration steps.

To investigate the convergence of these iterative calculations, the error of the iteration is computed for given emissivity values of tungsten. To get realistic data typical parameters of the setup are used: a laser pulse energy of 5 J at a wavelength of 1064 nm and a tungsten sample with a mass of 5 g. The values for the temperature-dependent specific heat capacity were taken from the literature [21], and the emissivity values are taken from Latyev et al. [22] and interpolated. According to this literature, for example, at a temperature of 2000 K, the emissivity at the desired wavelength of 1064 nm is $\varepsilon = 0.36$ and the specific heat capacity is $c_p = 175\,\mathrm{J \cdot kg^{-1} \cdot K^{-1}}$ for the hypothetic tungsten sample. From these temperature-dependent values, the resulting temperature rise ΔT is calculated according to Eq. 4. From Planck's law of thermal radiation result the virtually measured temperatures, T_M and ΔT_M, which are the starting points for the iteration. Figure 3 shows the relative difference of the calculated emissivity with respect to the starting literature value for each iteration step.

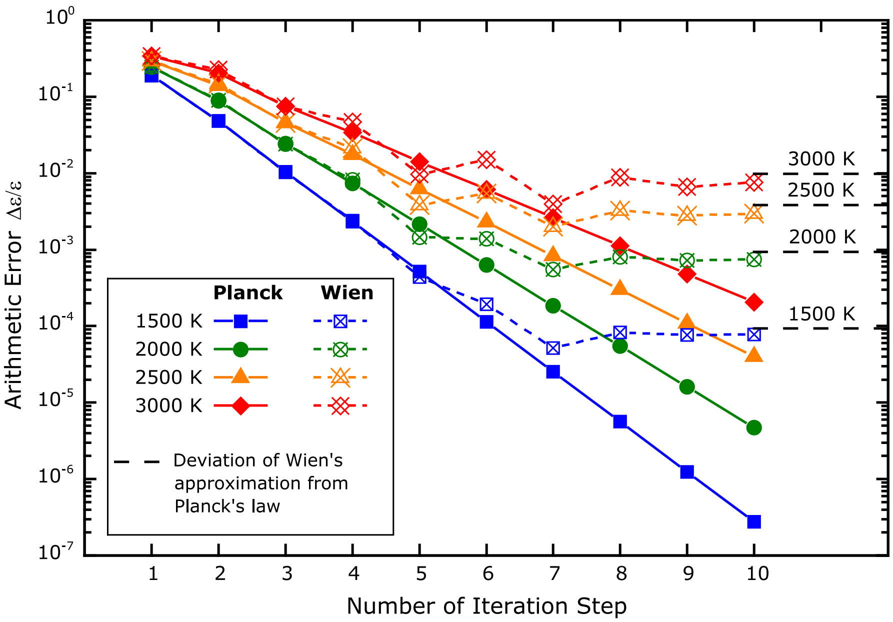

Fig. 3 Arithmetic error for the iterative calculation of the emissivity of tungsten. Calculation was performed for typical setup parameters: laser energy 5 J, wavelength of laser and radiation thermometer 1064 nm, and sample mass 5 g. The iteration of the temperature rise with the previous calculated emissivity value is done for the different furnace temperatures in two ways, explicit with Wien's approximation (*dashed lines*) and by solving an implicit equation according to Planck's law of thermal radiation (*solid lines*). The labels at the right scale indicate the inherent error of a temperature rise calculated with Wien's approximation at 1064 nm and the respective temperature values

Figure 3 shows good convergence of the iteration with Planck's law, where the relative error with respect to the given emissivity decreases exponentially. The error of the explicit calculations with Wien's approximation stabilizes after five to seven iteration steps. In both cases, the speed of convergence depends on the factor $\lambda_0 T$ and, for a given λ_0, is faster for smaller T values. The comparison with the inherent error of Wien's approximation (shown on the right side of Fig. 3) reveals that the error of the iteration stabilizes at the inherent error of Wien's approximation.

This analysis has proven that even with only a few iteration steps and the explicit recalculation of the temperature rise according to Wien's approximation (see Eqs. 8 and 14), the arithmetic error of the calculated emissivity is $<1\,\%$ for a measurement wavelength of 1064 nm up to furnace temperatures of 3000 K for tungsten. For materials with higher emissivity values, e.g., carbon, the arithmetic error becomes even smaller.

4.2 Implicit Equation

An alternative way to overcome the problem that the measured temperature rise itself depends on the emissivity, as described in the beginning of Sect. 4, is to derive an implicit equation for the thermodynamic temperature T_0 of the sample before receiving the laser pulse. To calculate this temperature, first the signal of the radiation thermometer—the photodiode current I_{Pd}—is considered. With Eqs. 6 and 11, it can be written as

$$\begin{aligned} I_{\mathrm{Pd}}(T) &= \int s(\lambda) L_\lambda(\lambda, T)\,\mathrm{d}\lambda = \int s_0 \delta(\lambda - \lambda_0)\varepsilon(\lambda, T) L_{\lambda,\mathrm{BB}}(\lambda, T)\,\mathrm{d}\lambda \\ &= s_0 \varepsilon(\lambda_0, T) L_{\lambda,\mathrm{BB}}(\lambda_0, T) \\ &= s_0 \varepsilon(\lambda_0, T) \frac{c_1}{\lambda_0^5}\left[\exp\left(\frac{c_2}{\lambda_0 T}\right) - 1\right]^{-1}. \end{aligned} \tag{15}$$

From the temperature-dependent change of the photodiode current around λ_0, T_0, an equation can be derived for the infinitesimal temperature rise $\mathrm{d}T_0$

$$\begin{aligned} \left.\frac{\mathrm{d}I_{\mathrm{Pd}}}{\mathrm{d}T}\right|_{\lambda_0, T\in[T_0, T_0+\Delta T]} &= s_0\varepsilon_0\left(-\frac{c_1}{\lambda_0^5}\right)\left[\exp\left(\frac{c_2}{\lambda_0 T}\right) - 1\right]^{-2} \exp\left(\frac{c_2}{\lambda_0 T}\right)\left(-\frac{c_2}{\lambda_0 T^2}\right) \\ &= \underbrace{s_0\varepsilon_0 \frac{c_1}{\lambda_0^5}\left[\exp\left(\frac{c_2}{\lambda_0 T}\right) - 1\right]^{-1}}_{I_{\mathrm{Pd},0} \equiv I_{\mathrm{Pd}}(\lambda_0, T\in[T_0, T_0+\Delta T])} \frac{\frac{c_2}{\lambda_0 T^2}\exp\left(\frac{c_2}{\lambda_0 T}\right)}{\exp\left(\frac{c_2}{\lambda_0 T}\right) - 1} \\ \mathrm{d}T_0 &= \frac{\mathrm{d}I_{\mathrm{Pd},0}}{I_{\mathrm{Pd},0}} \frac{\exp\left(\frac{c_2}{\lambda_0 T}\right) - 1}{\frac{c_2}{\lambda_0 T^2}\exp\left(\frac{c_2}{\lambda_0 T}\right)}. \end{aligned} \tag{16}$$

Due to Planck's law of thermal radiation, the temperature rise is proportional to the relative change of the photodiode current around λ_0, T_0.

The infinitesimal temperature rise from Eq. 16 is equal to the temperature rise from Eq. 4, which leads in connection with Eq. 15 to an implicit equation for the furnace temperature T_0

$$\begin{aligned} \mathrm{d}T|_{\lambda_0, T\in[T_0, T_0+\Delta T]} &= \mathrm{d}T_0 \\ \varepsilon_0 \frac{\mathrm{d}E}{mc_{p,0}} &= \frac{\mathrm{d}I_{\mathrm{Pd},0}}{I_{\mathrm{Pd},0}} \frac{\exp\left(\frac{c_2}{\lambda_0 T}\right) - 1}{\frac{c_2}{\lambda_0 T^2}\exp\left(\frac{c_2}{\lambda_0 T}\right)} \\ \frac{I_{\mathrm{Pd}}(T_0)}{s_0}\frac{\lambda_0^5}{c_1}\left[\exp\left(\frac{c_2}{\lambda_0 T}\right) - 1\right]\frac{\mathrm{d}E}{mc_{p,0}} &= \frac{\mathrm{d}I_{\mathrm{Pd},0}}{I_{\mathrm{Pd},0}} \frac{\exp\left(\frac{c_2}{\lambda_0 T}\right) - 1}{\frac{c_2}{\lambda_0 T^2}\exp\left(\frac{c_2}{\lambda_0 T}\right)} \\ \sqrt{\frac{I_{\mathrm{Pd},0}}{s_0}\frac{I_{\mathrm{Pd},0}}{\mathrm{d}I_{\mathrm{Pd},0}}\frac{\lambda_0^5}{c_1}\frac{\mathrm{d}E}{mc_{p,0}}\frac{c_2}{\lambda_0}\exp\left(\frac{c_2}{\lambda_0 T}\right)} &= T. \end{aligned} \tag{17}$$

This equation is solved for the temperature of the sample $T = T_0$ before the laser pulse heats the sample. For a real, non-infinitesimal event, the infinitesimal variables transform into the actual measured values of the laser pulse energy E_0 and the rise of the photodiode current ΔI_{Pd}

$$\begin{aligned} \mathrm{d}E &\to E_0, \\ \mathrm{d}I_{\mathrm{Pd},0} &\to \Delta I_{\mathrm{Pd}}. \end{aligned} \tag{18}$$

Finally, an implicit equation for the sample temperature T_0 directly before the laser pulse is derived, containing only known values

$$\sqrt{\frac{I_{\mathrm{Pd},0}}{\Delta I_{\mathrm{Pd}}}\frac{E_0 I_{\mathrm{Pd},0}}{s_0}\frac{c_2\lambda_0^4}{c_1 m c_{p,0}}}\exp\left(\frac{c_2}{2\lambda_0}\frac{1}{T_0}\right) = T_0. \tag{19}$$

The emissivity at this temperature and at the laser wavelength λ_0 results from the measured spectral radiance and Planck's law of thermal radiation according to

$$\varepsilon(\lambda_0, T_0) = \frac{L_\lambda^{\mathrm{Sample}}(\lambda_0)}{L_{\lambda,\mathrm{BB}}(\lambda_0, T_0)} = \frac{I_{\mathrm{Pd},0}/s_0}{L_{\lambda,\mathrm{BB}}(\lambda_0, T_0)}. \tag{20}$$

Although this looks like an exact solution for the problem described with Eqs. 4 and 5, it is based on an approximation. The replacement of the infinitesimal variables dE and d$I_{\mathrm{Pd},0}$ by the corresponding non-infinitesimal measured values causes an arithmetic error, which is calculated again with typical measurement parameters for a tungsten sample (refer to Sect. 4.1).

The origin of the arithmetic error becomes also apparent by the fact that the error increases with the laser energy and the resulting temperature rise, as shown in Fig. 4.

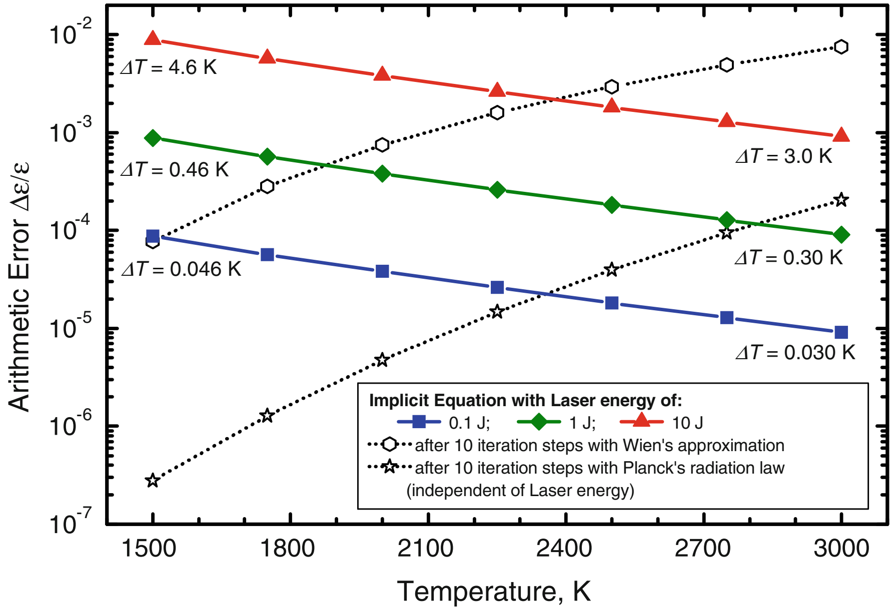

Fig. 4 Arithmetic error for the implicitly calculated emissivity of tungsten. The calculations are done for typical parameters of the experimental setup: wavelength of laser and radiation thermometer 1064 nm and a tungsten sample with a mass of 5 g. The implicit Eq. 19 is solved for different pulse energies and temperature values (*solid lines*). The corresponding values of the temperature rise are shown below the curves. The curves for the iterative calculations (*dashed lines*) are also given for comparison. They show the arithmetic error after 10 iteration steps with Wien's approximation and Planck's law of thermal radiation, respectively, which is independent of the laser energy (see Sect. 4.1)

Note that the error increases directly proportional with the laser energy, which influences also the rise of the photodiode current. However, a laser energy of 0.1 J is not practicable due to the low temperature rise, but for the values of 0.1 J, 1 J, and 10 J, the proportionality is obvious. Even for a pulse energy of 10 J, which causes a temperature rise of around 4 K for the used 5 g tungsten sample, the arithmetic error is below 1 %. In contrast, the arithmetic error of the iteratively computed emissivity is nearly independent of the laser energy, because in this case the error results from the iteration or Wien's approximation. The curves for the arithmetic error after 10 iteration steps are also given in Fig. 4 for comparison.

5 Discussion and Outlook

In this work, the feasibility of measuring the spectral emissivity with a dynamic method using a laser-flash apparatus is discussed. Two ways have been proposed to solve the problem, that the temperature rise, which is required to calculate the emissivity, itself depends on the unknown emissivity. The arithmetic error of the different methods has been discussed on the basis of a tungsten sample and emissivity values taken from the literature. Depending on the temperature, the arithmetic error of the simple and explicit iteration using Wien's approximation is less than 0.01 % to 1 %. Also, the arithmetic error of the implicit equation due to the non-infinitesimal variables is in this range, depending on the laser energy. Therefore, these arithmetic errors are below currently available measurement uncertainties. The iteration with Planck's law of thermal radiation, with an implicit equation in each iteration step, offers, in principle, an infinite small arithmetic error. The sensitivity of the described calculation methods with respect to measurement uncertainties have to be investigated further to choose the best suitable algorithm. On the basis of the mathematical results of this investigation, the laser flash method, which is up to now widely used to measure mainly the thermal diffusivity can be advanced by absolute measurement of the laser energy and the temperature rise to additionally measure the emissivity, if the specific heat capacity is known or vice versa. On the basis of the above detailed theoretical calculations, it seems now possible to measure the most interesting thermophysical material properties at high temperatures with one fast measurement setup, enabling a nearly complete thermal characterization of a sample with one measurement as already proposed nearly half a century ago.

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